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ACTIVE SHIELDING AND CONTROL OF ENVIRONMENTAL NOISE*

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In the framework of the research project supported by NASA under grant # NAG–1–01064, we have studied the mathematical aspects of the problem of active control of sound, i.e., time-harmonic acoustic disturbances. The foundations of the methodology are described in our paper [1]. Unlike many other existing techniques, the approach of [1] provides for the exact volumetric cancellation of the unwanted noise on a given predetermined region of space, while leaving unaltered those components of the total acoustic field that are deemed as friendly. The key finding of the work is that for eliminating the unwanted component of the acoustic field in a given area, one needs to know relatively little; in particular, neither the locations nor structure nor strength of the exterior noise sources need to be known. Likewise, there is no need to know the volumetric properties of the supporting medium across which the acoustic signals propagate, except, maybe, in a narrow area of space near the perimeter of the protected region. The controls are built based solely on the measurements performed on the perimeter of the domain to be shielded; moreover, the controls themselves (i.e., additional sources) are concentrated also only on or near this perimeter. Perhaps as important, the measured quantities can refer to the total acoustic field rather than to its unwanted component only, and the methodology can automatically distinguish between the two. In [1], we have constructed the general solution for controls. The apparatus used for deriving this general solution is closely connected to the concepts of generalized potentials and boundary projections of Calderon's type. For a given total wave field, the application of a Calderon's projection allows one to definitively tell between its incoming and outgoing components with respect to a particular domain of interest, which may have arbitrary shape. Then, the controls are designed so that they suppress the incoming component for the domain to be shielded or alternatively, the outgoing component for the domain, which is complementary to the one to be shielded.

In a number of subsequent publications (some of which are still in progress), we have looked into several related issues, including discrete control sources, surface and volumetric controls, and, most important, optimization of the control sources with respect to different criteria. In [5], we have constructed special types of discrete surface control sources that correspond to the continuous densities of the single- and double-layer potentials. The papers [4, 2, 3] are devoted to various aspects of optimization. There is a multitude of possible criteria for optimality that one can use. In many practical problems the cancellation of noise is only approximate and as such, the key criterion for optimization (or sometimes, the key constraint) is the quality of this cancellation, i.e., the extent of noise reduction. In contradistinction to that, we have considered ideal, or exact, cancellation so that every particular control field from the class that we identify would completely eliminate the unwanted noise on the domain of interest. Consequently, the criteria for optimality of the controls that we could employ did not include the level of the residual noise as a part of the corresponding function of merit, and rather depended only on the control sources themselves.

We have considered optimization of the acoustic source strength, optimization in the sense of least squares, and optimization of power. Optimization of the acoustic source strength, see [2], mathematically translates into the minimization of complex-valued functions in the sense of L_1 with conical constraints, which are only "marginally" convex. The corresponding numerical optimization problem appears very challenging

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even for the most sophisticated state-of-the-art methodologies, and even when the grid dimensions are small and the waves are long. Our central result in [2] is that the global L_1-optimal solution can, in fact, be obtained without solving the numerical optimization problem. This solution is given by a special layer of monopole-type sources on the perimeter of the protected region. We provide a rigorous proof of the global L_1-minimality in the one-dimensional case. We also provide numerical evidence that corroborates our result in the two-dimensional case, when the protected domain is a cylinder. Even though we cannot prove it yet, we believe that the result is correct in the general case as well, i.e., for multi-dimensional settings that include domains of arbitrary shape. We formulate it as a conjecture in the end of the paper [2].

In [4], we have optimized the controls in the sense of the least squares, or in other words, optimized the L_2 norm of the control sources. The latter criterion does not have such a transparent physical meaning as the L_1 norm does, but the minimum is easier to compute numerically, including the cases that involve additional equality type constraints originating form geometric definitions. In [3], we have minimized the power required for the operation of the control system. It turns out that the corresponding analysis necessarily involves interaction between the sources of sound and the surrounding acoustic field, the phenomenon that is often referred to as load by the field on the sources. Our key finding that we report in [3] may first seem counterintuitive, but in fact one can build a control system that would require no power input for operation and would even produce a net power gain while providing for the exact noise cancellation. This, of course, comes at the expense of having the original sources of noise to produce even more energy.

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